Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel Level 3 GCE

Centre Number	Candidate Number

Paper reference **8FM0/26**

Further Mathematics

Advanced Subsidiary
Further Mathematics options
26: Further Mechanics 2
(Part of option J)

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator allowed by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- You should show sufficient working to make your methods clear.
- Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8\,\text{m}\,\text{s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- The total mark for this part of the examination is 40. There are 4 questions.
- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- Good luck with your examination.

Turn over ▶

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Pearson

1.

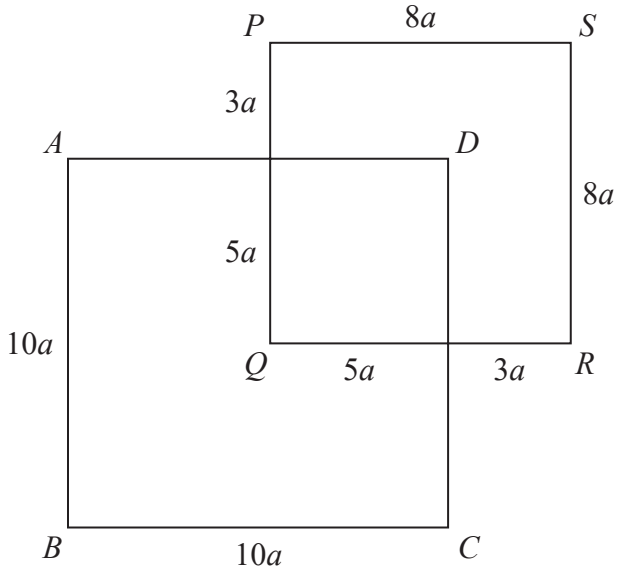


Figure 1

A uniform rod of length $72a$ is cut into pieces. The pieces are used to make two rigid squares, $ABCD$ and $PQRS$, with sides of length $10a$ and $8a$ respectively. The two squares are joined to form the rigid framework shown in Figure 1.

The squares both lie in the same plane with the rod AB parallel to the rod PQ.

Given that

- AD cuts PQ in the ratio $3:5$
- DC cuts QR in the ratio $5:3$

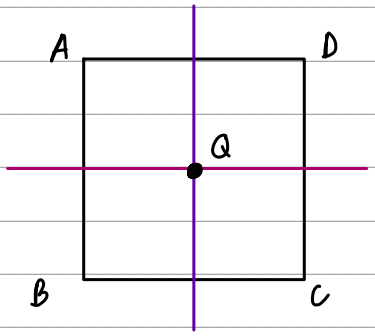
(a) explain why the centre of mass of square $ABCD$ is at Q.

(1)

(b) Find the distance of the centre of mass of the framework from B.

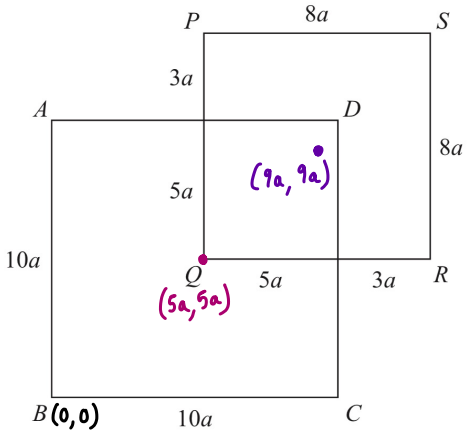
(5)

a) By symmetry, COM at centre of square which is at Q.
This is because Q is where the perpendicular bisectors of each side intersect with one another, which is the exact centre of the lamina.



Question 1 continued

b)



Let B be the origin

Looking at ABCD and PQRS as separate shapes

	ABCD	PQRS
COM:	(5a, 5a)	(9a, 9a)
Mass Ratios:	40	32

↳ the mass is proportional to the lengths of the rods

Overall COM of lamina from B

Length is proportional to the mass as the rod is uniform, we can substitute the force due to the mass with the area.

moments = force x perpendicular distance
The sum of moments is equal to the overall moment acting through the COM.
Mathematically → $\sum m_i x_i = \bar{x} \sum m_i$
Where m = force
x = perpendicular distance

Overall COM:

$$40\begin{pmatrix}5a\\5a\end{pmatrix} + 32\begin{pmatrix}9a\\9a\end{pmatrix} = 72\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix}$$

$$\begin{pmatrix}200a\\200a\end{pmatrix} + \begin{pmatrix}288a\\288a\end{pmatrix} = \begin{pmatrix}72\bar{x}\\72\bar{y}\end{pmatrix}$$

$$\begin{pmatrix}488a\\488a\end{pmatrix} = \begin{pmatrix}72\bar{x}\\72\bar{y}\end{pmatrix}$$

$$\begin{pmatrix}\bar{x}\\\bar{y}\end{pmatrix} = \begin{pmatrix}\frac{61}{9}a\\\frac{61}{9}a\end{pmatrix}$$

DISTANCE of COM from B

↙ Use Pythagoras' theorem on $\bar{x}$ and $\bar{y}$

$$= \sqrt{\left(\frac{61}{9}a\right)^2 + \left(\frac{61}{9}a\right)^2}$$

$$= \frac{61\sqrt{2}}{9}a$$

(Total for Question 1 is 6 marks)

2.

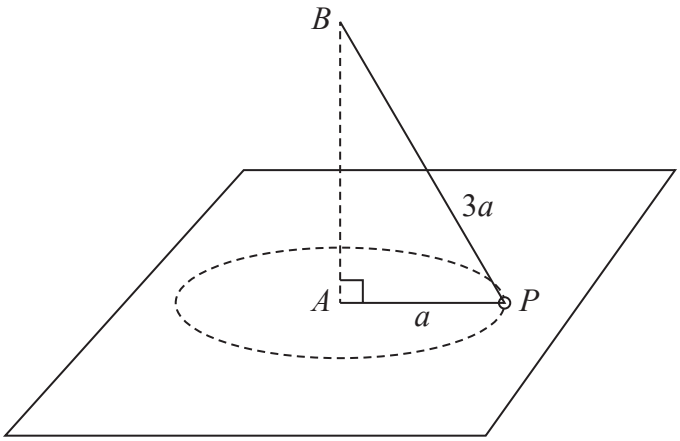


Figure 2

A small smooth ring P, of mass m, is threaded onto a light inextensible string of length $4a$.
One end of the string is attached to a fixed point A on a smooth horizontal table.
The other end of the string is attached to a fixed point B which is vertically above A.
The ring moves in a horizontal circle with centre A and radius a, as shown in Figure 2.

The ring moves with constant angular speed $\sqrt{\frac{2g}{3a}}$ about AB.

The string remains taut throughout the motion.

(a) Find, in terms of m and g, the magnitude of the normal reaction between P and the table.

(6)

The angular speed of P is now gradually increased.

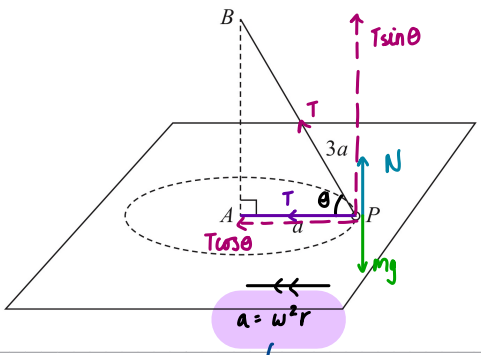
(b) Find, in terms of a and g, the angular speed of P at the instant when it loses contact with the table.

(3)

(c) Explain how you have used the fact that P is smooth.

(1)

a)



↳ memorised formulae

① Tension in either side of the string is equal because P is smooth ↘

$T = T$

$\cos\theta = \frac{a}{3a} = \frac{1}{3}$

$\theta = \cos^{-1}\left(\frac{1}{3}\right)$

$\sin\theta = \frac{2\sqrt{2}}{3}$

Resolving forces horizontally

$R(\rightarrow)$: $T + T\cos\theta = m\left(\sqrt{\frac{2g}{3a}}\right)^2 \times a$ ← from question

$T + T\cos\theta = \frac{2mg}{3}$

$T + \frac{1}{3}T = \frac{2mg}{3}$

$\frac{4}{3}T = \frac{2mg}{3}$

$4T = 2mg$

$T = \frac{1}{2}mg$

Resolving forces vertically

$R(\uparrow)$: $T\sin\theta + N = mg$

$\frac{1}{2}mg\left(\frac{2\sqrt{2}}{3}\right) + N = mg$

$N = mg - \frac{\sqrt{2}}{3}mg$

$N = mg\left(1 - \frac{\sqrt{2}}{3}\right)$

b) At max speed, at the instant when it loses contact with the table $\Rightarrow N = 0$

Applying new value of N

NEW Resolving vertically

$R(\uparrow)$: $T\sin\theta = mg$

$\frac{2\sqrt{2}}{3}T = mg$

$T = \frac{3\sqrt{2}}{4}mg$

NEW Resolving horizontally

$R(\leftarrow)$: $\frac{4}{3}T = m\omega^2 a$

$\omega^2 = \frac{4T}{3ma}$

$\omega = \frac{4\left(\frac{3\sqrt{2}}{4}mg\right)}{3ma}$

$\omega^2 = \frac{g\sqrt{2}}{a}$

$\omega = \sqrt{\frac{g\sqrt{2}}{a}}$

Question 2 continued

c) Tension in the string is the same on either side of P

Question 2 continued

(Total for Question 2 is 10 marks)

3.

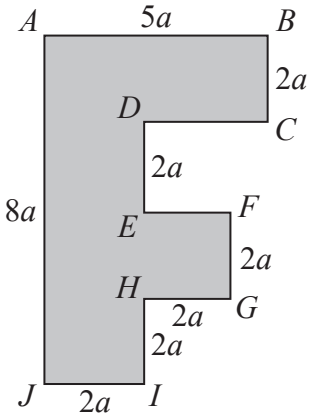


Figure 3

The uniform lamina $ABCDEFGHIJ$ is shown in Figure 3.

The lamina has $AJ = 8a$, $AB = 5a$ and $BC = DE = EF = FG = GH = HI = IJ = 2a$.

All the corners are right angles.

(a) Show that the distance of the centre of mass of the lamina from AJ is $\frac{49}{26}a$

(5)

A light inextensible rope is attached to the lamina at A and another light inextensible rope is attached to the lamina at B. The lamina hangs in equilibrium with both ropes vertical and AB horizontal. The weight of the lamina is W.

(b) Find, in terms of W, the tension in the rope attached to the lamina at B.

(3)

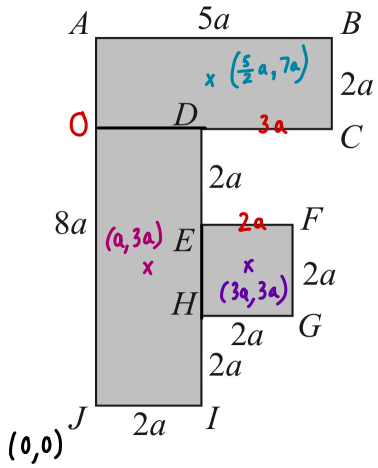
The rope attached to B breaks and subsequently the lamina hangs freely in equilibrium, suspended from A.

(c) Find the size of the angle between AJ and the downward vertical.

(5)

Question 3 continued

a)



- Splitting the lamina into 3 shapes

	JODI	EFGH	ABOC
Area	$12a^2$	$4a^2$	$10a^2$
Mass ratio :	6	2	5
COM from AJ	a	$3a$	$\frac{5}{2}a$

COM Total : $6(a) + 4(3a) + 5\left(\frac{5}{2}a\right) = 13\bar{x}$

$\frac{49}{2}a = 13\bar{x}$

$\bar{x} = \frac{49a}{26}$ from AJ

Overall COM of lamina from AJ

Since it is from AJ, only consider x-axis.

Area is proportional to the mass as the lamina is uniform, we can substitute the force due to the mass with the area.

moments = force x perpendicular distance

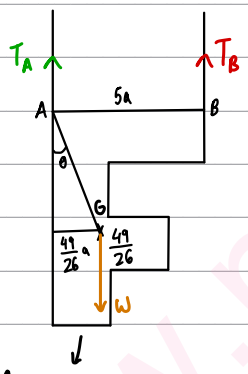
The sum of moments is equal to the overall moment acting through the COM.

Mathematically → $\sum m_i x_i = \bar{x} \sum m_i$

Where m = force

x = perpendicular distance

b)

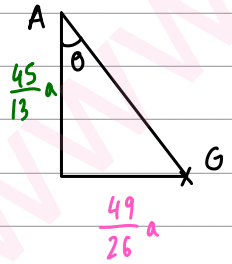


Taking moments about A

$5a \times T_B = \frac{49}{26}a \times W$

$T_B = \frac{49}{130}W$

c)



To find COM IJ (following same steps as $\bar{x}$ but with $\bar{y}$)

$12(3a) + 4(3a) + 10(7a) = 26\bar{y}$

$118a = 26\bar{y}$

$\bar{y} = \frac{59}{13}a$

FROM AB: $8a - \bar{y} = 8a - \frac{59}{13}a = \frac{45}{13}a$

$\tan\theta = \frac{\frac{49}{26}a}{\frac{45}{13}a} = \frac{49}{90}$

$\theta = \tan^{-1}\left(\frac{49}{90}\right) = 28.6°$

$\therefore \theta = 28.6°$ (3sf)

Question 3 continued

Question 3 continued

(Total for Question 3 is 13 marks)

4. A particle P moves on the x-axis. At time t seconds, $t \geqslant 0$, P is x metres from the origin O and moving with velocity $v\,\text{m s}^{-1}$ in the direction of x increasing, where

$$v = 5\sin 2t$$

When $t = 0$, $x = 1$ and P is at rest.

(a) Find the magnitude and direction of the acceleration of P at the instant when P is next at rest.

(4)

(b) Show that $1 \leqslant x \leqslant 6$

(4)

(c) Find the total time, in the first 4π seconds of the motion, for which P is more than 3 metres from O

(3)

a) $a = \frac{dv}{dt}$ $t=0,\ x=1,\ v=0$ (Boundary conditions)

$a = \frac{d(5\sin 2t)}{dt}$

$a = 10\cos 2t$

Next at rest when $v=0$

$0 = 5\sin 2t$

$0 = \sin 2t$

$t_1 = 0,\ t_2 = \frac{\pi}{2}$

$t = \frac{\pi}{2}$, $a = 10\cos\left(2\left(\frac{\pi}{2}\right)\right) = -10\,\text{ms}^{-2}$

$\Rightarrow$ acceleration of P has magnitude $10\,\text{ms}^{-2}$ in the direction of O

b) $x = \int v\,dt$

$x = \int 5\sin 2t\,dt$

$x = -\frac{5}{2}\cos 2t + C$

Using boundary conditions $t=0,\ x=1$

$1 = -\frac{5}{2} + C$

$C = \frac{7}{2}$

$\therefore x = \frac{7}{2} - \frac{5}{2}\cos(2t)$

The boundaries of function $\cos(2t)$ are

$-1 \leqslant \cos 2t \leqslant 1$

so $\frac{7}{2} - \frac{5}{2}(1) \leqslant \frac{7}{2} - \frac{5}{2}\cos 2t \leqslant \frac{7}{2} - \frac{5}{2}(-1)$

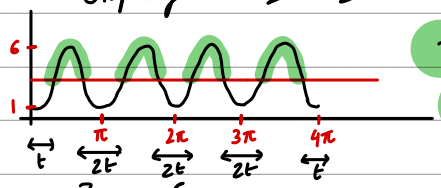
$1 \leqslant x \leqslant 6$

sides switch because you are multiplying by a negative number ('$-\frac{5}{2}$')

→ Plot on graphing calculator for ease

Question 4 continued

c) Graph of $x = \frac{7}{2} - \frac{5}{2}\cos 2t$



$x = \frac{7}{2} - \frac{5}{2}\cos 2t$

Time for which P is more than 3m away from O

$T = 4\pi - 8t$

$= 4\pi - 8(0.6847\ldots)$

$T = 7.1\text{s}$

When $x = 3$, $3 = \frac{7}{2} - \frac{5}{2}\cos 2t$

$\frac{1}{5} = \cos 2t$

$t = \frac{1}{2}\cos^{-1}\left(\frac{1}{5}\right)$

$t = 0.6847\ldots\text{s}$

Question 4 continued

Question 4 continued

Question 4 continued

(Total for Question 4 is 11 marks)

TOTAL FOR FURTHER MECHANICS 2 IS 40 MARKS